NUMERICAL ANALYSES OF WESTFALL MANUFACTURING 42 INCH STATIC MIXING DEVICE

1.0 INTRODUCTION

Westfall Manufacturing contracted the Alden Research Laboratory, Inc. (ALDEN) to calculate the flow-induced stress distribution on a proprietary 42 inch diameter static mixing plate to optimize plate thickness. The effort included a combined Computational Fluid Dynamic (CFD) analysis and structural Finite Element Analysis (FEA) of the plate to calculate the normal and tangential shear stress distribution within the plate and to model the deflection of the plate in response to the flow field. The scope of work included analyses of three unique plate thicknesses. The three dimensional CFD simulation was performed to simulate the flow through the mixer and a FEA analysis (coupled with the CFD analysis) was used to calculate the resulting time averaged stress/strain distribution within the mixing plate as well as the time average mixing plate deflection. To confirm the linearity of the plate stress/strain relationship, three flow rates were analyzed for one plate thickness covering the expected operating flow range.

2.0 DEVELOPMENT OF THE NUMERICAL MODEL

The commercial finite element-based fluid flow analysis program FIDAP V8.5 (Fluent, Inc.) was used to perform three dimensional computer modeling of the flow patterns through the mixing plate. FIDAP is a state-of-the-art computer code for simulating both internal and external turbulent flows and is especially well suited for the present analysis due to its ability to model the parameters necessary for performing fluid structure interaction calculations. In addition to FIDAP, the FEA code COSMOS/DesignSTAR was used to perform the structural analyses. The complex three dimensional computational mesh was developed using the grid generating program GAMBIT 1.3 (Fluent, Inc.). Solution analyses were performed using the post-processing software FIELDVIEW (Intelligent Light, Inc.).

The modeled geometry extended two diameters upstream and five diameters downstream of the mixing plate. A fully developed velocity profile with a prescribed turbulence intensity was specified at the upstream boundary. The no-slip (zero velocity) boundary condition was applied along all solid surfaces. A material property set was applied to the mixing plate to properly model flow-induced deflections.

A total of three mixing plate thicknesses were analyzed at one selected flow (typical operating flow rate) and one selected plate thickness was analyzed for three flows covering the expected operating range. The plate thicknesses analyzed included 0.375, 0.5 and 0.625 in. with the 0.625 in. geometry analyzed at three flow rates (4,166, 31,166 and 37,166 gpm). The typical operating flow rate chosen (based on discussions with Westfall Manufaturing personnel) for the simulations was 31,166 gpm (corresponding to an average pipe velocity of approximately 7.2 ft/s).

3.0 RESULTS

The flow simulations were performed for the 42 in. diameter mixer plate geometry shown in Figure 1. The simulated geometry extended one pipe diameter upstream and five diameters downstream of the mixer plate to ensure that the numerical boundary conditions did not influence the flow variables in the region of interest near the plate. Figure 2 shows a portion of the computational grid. Flow streamlines are shown in Figure 3 for one quartile of the flow domain. The flow patterns in each of the four quartiles were assumed similar. Note the well-defined wake behind the mixing plate. Velocity vectors and speed (by color) on a horizontal plane are shown in Figure 4.

The calculated flow field illustrated in Figures 3 and 4 was used to predict the stress distribution and deflection of the mixing plate using the finite element analysis (FEA) software program COSMOS/DesignSTAR (by Structural Research and Analysis Corp.). A summary of the maximum mixer plate deflection, as a function of plate thickness, is given in Table 1. These simulations were performed for a total flow of 31,166 gpm.

Table 1

42 inch Mixer; Flow = 31,166 gpm, Pipe Velocity = 7.2 ft/s

Plate Thickness (in)	Maximum Deflection (in)
0.625	0.002
0.500	0.005
0.375	0.033

The stress distribution and deflection for each plate thickness at a flow of 31,166 gpm is illustrated in Figures 4 through 10. The stress and deflection distributions (color-coded) are similar in all cases, however, close inspection of the scale on each figure reveals that the magnitudes of the stresses and deflections differ. A common scale between the figures was not used since this would result in a uniform color-coded contour plot for the thicker geometries. Figures 11 through 14 show the Von Mises stress distribution and deflection (Uz) for the 0.625 in. mixer plate at flow rates of 4166 gpm and 37166 gpm, respectively. The maximum deflection of the 0.625 in. mixing plate at these flow rates are given below in Table 2.

Table 2

42 inch Mixer, 0.625 inch Thickness

Flow Rate(gpm)	Deflection (in)
4,166	0.00004
37,166	0.0035

The plate natural frequencies and minimum factor of safety, in terms of yielding, were predicted as shown in Table 3.

Table 3

Summary of FEA Results of the 42 Inch Mixer Plate

Pipe Velocity (ft/s)	Average loading (psi)	Maximum deflection (in)	Maximum Plate Stress (psi)	Natural Frequency (hz)	Minimum Factor of Safety (yield)
5/8 Inch Thick Plate[1]					
8.61	0.40	0.004	481 (at 8.61 ft/s)	107	51 (at 8.61 ft/s)
7.22	0.28	0.002			
0.97	0.005	<0.0001			
½ Inch Thick Plate[1]					
8.61	0.40	0.007	1125 (at 8.61 ft/s)	87	22 (at 8.61 ft/s)
7.22	0.28	0.005			
0.97	0.005	<0.0001			
3/8 Inch Thick Plate[1]					
8.61	0.40	0.047	3300 (at 8.61 ft/s)	46	5 (at 8.61 ft/s)
7.22	0.28	0.033			
0.97	0.005	0.0006			

The analysis assumed ferritic stainless steel with ultimate strength (S_{ut}) = 60,000 psi, and yield strength (S_y) = 25,000 psi[1].

The maximum plate stress was used to estimate the expected cyclic life of the 3 mixer plates based on the following equation. Conservatively, it was assumed that the maximum stress above would be completely reversed over the each cycle of the plate and over the entire life of the plate. The number of completely reversed loadings that the material can withstand before failure can be estimated [1] as follows:

$$N = (\sigma_a/a)^{1/b} \qquad (1)$$

Where σ_a = the applied stress (from Table 3),

$$a = (0.9S_{ut})^2/S_e \qquad (2)$$

$$b = -(1/3)\log(0.9S_{ut}/S_e) \qquad (3)$$

and the endurance limit, Se, is defied as

$$S_e = 0.5(S_{ut}) \qquad (4)$$

Because the predicted maximum stresses applied are well within the elastic range of the stainless steel, as indicated by the relatively high factors of safety in the table above, the cyclic life of these plates is quite high as shown in Table 4. An estimate of the expected life of the plates was made using the predicted natural frequency as a basis for the rate of the cyclic loading. It was found that the expected life of each plate well over a thousand years indicating that the effects of fatigue with the predicted loading of these mixer plates are insignificant.

Table 4

Plate thickness	Number of cyclic loadings to failure (N)
3/8	1.8E+17
1/2	5.7E+22
5/8	1.2E+27

[1]Mechanical Engineering Design/ J. E. Shigley, C. R. Mischke. Mcgraw-Hill pg. 280

The above analysis illustrates that under uniform, steady flow loading conditions each of the plates will not yield in a practical length of time. However, this steady-state analysis does not address the case where the mixer plate resonates at its natural frequency and a superposition of amplitudes may occur. In this scenario, the shedding frequency of the vorticies downstream of the plate may match the natural frequency of the mixer plate resulting in a superposition of amplitudes. This would progressively increase the maximum stress concentration at each load cycle and ultimately reduce the fatigue life of the plate. To determine if the plate resonates at its natural frequency a time-dependent CFD/FEA analyses must be performed. Alternately, a prototype mixer would be outfitted with strain gauges and flow tested in a suitable flow loop at the three flows under investigation.

4.0 CONCLUSIONS

Based on the results of the CFD and FEA simulations the following conclusions are drawn:

(1) The average deflection of each plate thickness simulated was small over the range of flows considered.

(2) Under uniform, steady flow loading conditions, none of the three plates considered yielded in a practical length of time.

(3) Further CFD and/or physical model studies would be required to determine if the mixing plate resonates at its natural frequency.